# Bachelor of Science (B.Sc.) Semester–IV Examination

## MATHEMATICS ($M_8$-Mechanics)

## Paper—II

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

## UNIT—I

1. (A) The moments of a given system of forces about three points (2, 0), (0, 2) and (2, 2) are 3, 4 and 10 units respectively. Find the magnitude of the resultant force and find the equation of its line of action. 6

(B) A regular hexagon ABCDEF consists of six equal rods which are each of weight W and are freely joined together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string, prove that its tension is $W/\sqrt{3}$. 6

**OR**

(C) Derive a Cartesian equation of the common catenary i.e. Derive $y = c \cosh (x/c)$. 6

(D) If a chain is suspended from two points A and B on the same level and depth of the middle point below AB is $(\ell/n)$, where $2\ell$ is the length of the chain, show that the horizontal span AB is equal to

$$\ell(n-1/n)\log\left(\frac{nH}{n-1}\right).$$ 6

## UNIT—II

2. (A) The velocity of a particle along and perpendicular to the radius vector are $\lambda r$ and $\mu\theta$. Find the path and show that the acceleration along and perpendicular to the radius vector are $(\lambda^2 r - \mu^2\theta^2)/r$ and $\mu\theta(\lambda + \frac{\mu}{r})$ respectively. 6

(B) A small bead slides with constant speed v on a smooth wire in the shape of the cardioid $r = a(1 + \cos\theta)$. Show that the value of $\frac{d\theta}{dt}$ is $\left(\frac{v}{2a}\right)\sec\left(\frac{\theta}{2}\right)$ and that the radial component of the acceleration is constant. 6

**OR**

(C) A particle describes a curve, for which s and x vanish simultaneously, with uniform speed v. If the acceleration at any point 's' be $\frac{v^2c}{s^2+c^2}$, find the intrinsic equation of the curve. 6

(D) Show that the particle executing simple Harmonic Motion requires $\frac{1}{6}$th of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude. 6

## UNIT—III

3. (A) Derive the Lagrange's equations of motion given by

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{q}_j}\right]-\left(\frac{\partial T}{\partial q_j}\right)=Q_j,\ j=1,2,\dots\dots\dots\ n$$

by using D'Alembert's principle,

where $Q_j=\sum_i \vec{F}_i.\frac{\partial \vec{r}_i}{\partial q_j} \equiv$ the components of the generalized force,

$T=\sum_i \frac{1}{2}m_i v_i^2 \equiv$ the kinetic energy of the system,

$q_j \equiv$ the generalized coordinates,

$\dot{q}_j \equiv$ the generalized velocities. 6

(B) A bead is sliding on a uniformly rotating wire in a force-free-space. If w is the angular velocity of rotation, then show that the equation of motion is given by $\ddot{r} = rw^2$. 6

**OR**

(C) Obtain the Lagrange equations of motion for a spherical pendulum i.e. a mass point suspended by a rigid weightless rod. 6

(D) For conservative system, prove that the Lagrange equations can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2, \ldots\ldots\ldots, n,$$

where $L \equiv$ Lagrangian for the system $= T - V$,

$T \equiv$ K.E. of the system,

$V \equiv$ P.E. of the system,

$q_j \equiv$ generalized coordinates,

$\dot{q}_j \equiv$ generalized velocities. 6

## UNIT—IV

4. (A) For a general system of mass points with position vectors $\vec{r}_i$ and applied forces $\vec{F}_i$, including any forces of constraint, prove that $\overline{T} = -1/2\overline{\sum_i \vec{F}_i \cdot \vec{r}_i}$, where T = kinetic energy of the system. 6

(B) Show that if a particle describes a circular orbit under the influence of an attractive central force directed toward a point on a circle, then the force varies as the inverse fifth power of the distance. 6



**OR**

(C) In a central force field, prove that the relation between r and t is given by

$$t=\int_{r_0}^{r}\frac{dr}{\sqrt{\left[\frac{2}{m}\left(E-V-\frac{\ell^2}{2mr^2}\right)\right]}},$$

where E ≡ total energy, V ≡ potential energy. 6

(D) Prove that the orbit of a particle moving in a central force field is given by

$$\theta=\theta_0-\int_{u_0}^{u}\frac{du}{\sqrt{\left[\frac{2mE}{\ell^2}-\frac{2mV}{\ell^2}-u^2\right]}}$$

where E ≡ total energy

V ≡ potential energy

m ≡ mass. 6

## UNIT—V

5. (A) Let X and Y be the sum of the resolved parts of the forces about $O_x$ and $O_y$ compounded into a single force R acting at the origin O, and G be their moment about O. If G' is the moment of the system about any point (z, η) and R = 0, then show that G ≠ 0. 1½

(B) For the common catenary, show that $y^2 = c^2 + s^2$. 1½

(C) A point describes the equiangular spiral $r = e^{\theta}$ with a constant angular velocity 'w' about O. Show that radial acceleration is zero. 1½

(D) A point moves in a plane curve, so that its tangential and normal accelerations are equal, show that $v = c\, e^{x}$. 1½

(E) If the total torque $\vec{N}$ is zero then prove that $\dot{\vec{L}} = 0$ and the angular momentum $\vec{L}$ is conserved. 1½

(F) Define degrees of freedom with examples for xy-plane. 1½

(G) In a central force field, prove that the areal velocity is constant. 1½

(H) Define a central orbit and write its differential equation. 1½